Understanding the Proof of the Collatz Conjecture

Version: 1.0
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1. Summary

With the proxy decomposition method presented in "Proof of the Collatz Conjecture" [01], a valuable model for describing a Collatz tree as a composition of forkfree terminated chains was found. This paper is a short lecture on the same theme and contains a more concentrated elaboration of the basic considerations. For that, the provided analysis-, ordering- and reduction-schemes for the Collatz tree [01] are skipped and the investigations starts directly with the reduced, odd numbered Collatz tree. A new Definition called "evolution chain" will be proposed characterizing the set of conditions needed for possibly any proof of the Collatz Conjecture. The paper confirms the principal correctness of the "Proof of the Collatz conjecture" [01] using slightly modified and terminologically enhanced methods.

2. Prerequisite

Only the odd-numbered part of collatz series will be used. That simply means that all even numbers generated with the standard "collatz generator" are ignored.

To produce the fork- or branch-free chains we are using the same rules as we used in [01] for the proxy decomposition of the tree. With a new Definition of the used chains we can use a simple to understand terminology and will discuss the propagating operations in "Collatz direction" only.

With an infinite tree it's more logical to start with small numbers, so we will get the chains near the trunk mainly at first. For that, the assembling of the tree will grow more or less in "tree direction" (from trunk to leafs).

3. Definition: Evolution Chain

An evolution chain is a series of natural numbers where each link is a bijective [02] mapping of it’s predecessor and where each link is unique within the series. An evolution chain is incapable to form rings with it's own links as well as with any other evolution chain within an universe of evolution chains formed with identical rules. To fulfill this challenge, at least one end of the chain has to contain a terminating link preventing any connection to another evolution chain. Such a prevention could be a diophantine limitation, a stop condition or out of linking skills. Example 1 with diophantine limitation:

\[
f(x) = \frac{2x - 1}{3}, \quad f_0 = 53 \quad \{x, f(x): 1, 3, 5, 7, \ldots\}
\]

\[
s(x): \quad 53 \quad 35 \quad 23 \quad 15 \quad 29/3?
\]

The series \( s(x) \) ends at \( f(x) = \frac{2 \cdot 15 - 1}{3} = 29/3 \), exceeding the definition range of natural numbers.

Example 2 with a limitation by rule:

\[
f(x) = \frac{3x + 1}{2}, \quad f_0 = 15, \text{ stop clause: } (x-5) \mod 8 = 0;
\]

\[
s(x): \quad 15 \quad 23 \quad 35 \quad 53 \quad (.)
\]

The series stops at (.) because the stop condition is reached by (53-5) \mod 8 = 0;

4. Methods: Using the the Proxy Decomposition Parameters
As mentioned before, we will propagate in "Collatz direction" to get any of the evolution chains. For that we need only a reduced part of the original flow chart (Fig. 1):

5.1 Creating the Building Blocks: the Evolution Chains

In contrast to the infinitely long even numbered branches within the original Collatz tree, the reduced odd numbered tree has (leaves) for any natural odd number divisible by 3 \([01]\), (see 2.1, Examle 1). Actually, both ends of the Evolution Chains will be defined:

**start condition (heads):** \(u \mod 3 = 0\) \(\{u = 1, 3, 5, 7, \ldots\}\)

The heads (red blocks in Fig. 2) of the chains will be 3, 9, 15, 21, 27, ...

The stop condition (gray blocks in Fig. 2) are given by Fig. 1:

**Stop condition (foots):** \((u-5) \mod 8 = 0\) \(\{u = 1, 3, 5, 7, \ldots\}\)

The flowchart of Fig. 1 is the cook book to generate the infinite number of Evolution Chains. How the operations are applied to the series can alternatively be shown in Fig. 2: Starting with a head link in the green column having a red marker on the left side, any following number can be found at the left column (numbers with blue background). For the next number, the blue number has to be found in the green column and the next number is found in the left column again. This procedure will be applied until a gray marker is visible in the green column. This (green) number is the last link of the Evolution Chain identified as foot link. The number with gray background at the left of such a foot link is not part of the actual Evolution Chain but shows the member link of another Evolution Chain where the foot link will be connected to.

The regularity of the alternating red and gray blocks can be interpreted as a confirmation of the correctness of the proxy decomposition as well as the foundation of Evolution Chains, which is the same as the "cut condition" of the proxy decompositions corresponds to the stop condition of Evolution Chains. But be aware, that we can't say anything about the lengths of the the Chains.
Using the operations flowchart Fig. 1, or using the evolution chain scheme, always starting with the head links of the chain led to the infinite set of evolution chains. Fig. 3 shows the first ... chains.

Fig 3: The 10 evolution chains with the lowest head values: 3, 9, 15, ...45, 51, 57. The numbers at the left of the chains are the assembly joint numbers.
5.2 How to get the Assembly Joint Numbers

With any odd number in the original Collatz tree a infinite number of even numbers is attached. This even numbers can be considers as a new generation of branches. About every second branch will then generate an odd number again [see 01]. The first daughter of every generation is nominated as prox for the whole generation... Because all single even numbers as well as all chains of even numbers are ignored, we can observe a gap between the foot of an evolution chain and its connecting link on its "last generation" branch. The calculation for finding the assembly joint number can be provided by simple applying the following operations on the foot element:

1. \( x(n+1) = 3x+1 \)
2. \( x(n+1) = x(n) \div 2 \)  repeat until \( x \) is an odd number

As mentioned above, the assembly joint numbers also can be found on the evolution chain scheme: all numbers in the gray field are the requested assembly joint numbers.

5.3 Assembling the Tree

Assembling the odd number Collatz tree is done by simly docking the foot elements of each chain to the corresponding assembly joint element given at the left of each chain in Fig. 3 or using the grey numbers from the evolution chain scheme. The second and the third chain e.g. can be connected with the link element number 5 of the first chain. Some of the chains are created long before their connecting parter will be created. Each assembly joint will collect an infinite number of evolution chains at the end of the assembly procedure.

The considerations can be extended to the original Collatz tree: all even numbers (the 0 is not used) can be inserted at the original place without any consequence. The attached even numbers, e.g. the attachement of the infinite even number series to the head link 9:

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.. 9  18  36  72  144  288 ...
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does not produce loops and therefore is also no danger for the proof. That's because the attached links do not have any linking capapilities.

6 Conclusions

The proof of the collatz conjecture is straightforward, because collatz odd number tree is loopless with only the exeption at 4, 2, 1, 4.... The loopless tree is proofed, as it had been broken completely into evolution chains defined above. The assembling of the tree does not deliver a contribution to the proof. It's the terminating head of each evolution chain turning out to be the key element for the proof of the collatz conjecture, capable to prevent any loops. In combination with the essential split at exactly defined locations, (proxi cut at the beginning of each generation), the proof is complete for the odd numbered Collatz tree as well as for the original Collatz structure.

The paper confirmed the principal correctness of the "Proof of the Collatz conjecture" [01] with slightly modified and terminologically enhanced methods.

The future will show if an even more compact proof or even a graphical proof can be found. A promising field of research will certainly continue to deliver great yet unknown results and applications.

7. Literature

[01]  FRANZ ZIEGLER: Proof of the collatz conjecture V1.2  www.collatz.com